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ANALYSIS OF FUEL CONSUMPTION FOR AN AIRCRAFT DEPLOYMENT WITH MULTIPLE AERIAL REFUELINGS

by
BRETT ALAN BUSH

A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

OPERATIONS RESEARCH

Raleigh

2006

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Thom J. Hodgson, Chair of Advisory Committee

ABSTRACT

BUSH, BRETT ALAN. Analysis of Fuel Consumption for an
Aircraft Deployment with Multiple Aerial Refuelings.
(Under the direction of Thom J. Hodgson.)

The purpose of the research has been to derive an

algorithm that finds optimal aerial refueling segments

(non-instantaneous) for a single aircraft deployment while

also accounting for atmospheric winds. There are two

decision variables: (1) Where to locate the refueling

segments? (2) How much fuel to offload at each refueling

segment? Later in the dissertation, a third decision

variable is explored: How much fuel to load onto the

aerial refueling aircraft?

In previous research, the problem of having a single

aircraft deployment with one instantaneous aerial refueling

has been explored and solved. This paper piggybacks on

that research and extends it. The first step (Problem P1)

is deriving an algorithm that finds the optimal aerial

refueling points for a single aircraft deployment with

multiple instantaneous aerial refuelings. In the next step

(Problem P2), one assumes aerial refueling is not

instantaneous (in an effort to make the problem and

solution more realistic), but requires some time frame

depending on an offload rate. In problem (P2), optimal

refueling segments are found (versus optimal refueling points). In the last problem (P3), one looks at a very similar algorithm that factors the winds aloft into the minimization algorithm. Finally, this paper looks at three distinct deployment scenarios with two aerial refuelings required. All of the scenarios were first planned by the U.S. Air Force and the results given to the author. Potential fuel and cost savings associated with using the aforementioned algorithms instead of current methods are then analyzed.

BIOGRAPHY

After graduating from high school at Adirondack Central School in Boonville, NY in 1993, Brett Bush attended the United States Air Force Academy in Colorado Springs, CO. In 1997, he graduated with a Bachelor of Science in Mathematical Science and was commissioned as a second lieutenant in the U.S. Air Force. His first assignment was to the 49th Test and Evaluation Squadron at Barksdale AFB, LA to perform analysis on U.S. Air Force bomber weapon systems. In 2000, he was reassigned to Hanscom AFB, MA where he worked on modeling, simulation, analysis, and systems integration of Air Force C4ISR systems. While at Hanscom AFB, Brett attended Northeastern University and obtained a Master of Science in Operations Research. In 2003, the Air Force reassigned him to Raleigh, NC to attend North Carolina State University and obtain a Doctor of Philosophy in Operations Research.

ACKNOWLEDGEMENTS

I would like to thank Dr. Thom Hodgson for his guidance and wisdom which have been second to none. It has been a privilege to work under him. I also would like to thank my other committee members, Dr. Jeffrey Thompson, Dr. Michael Kay, Dr. Russell King, and Dr. Yahya Fathi for their time and help. Finally, I would like to thank my wife. Without her love and support, I could never have completed this research and doctoral program.

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CHAPTER I

INTRODUCTION

Problem Description

In today's world, the ability to deploy aircraft, supplies, and manpower is an essential tenet to military power. Often, the deployment distance will be outside the range of the aircraft. This is usually overcome through the use of aerial refueling. Sometimes, more than one refueling will be required. This dissertation deals almost exclusively with that possibility (although the algorithm also works for one refueling).

The objective is to determine the optimal points on the earth in which to refuel during a deployment from an origin base to a destination with more than one refueling required. This will also produce another decision variable: the amount of fuel to offload at each refueling point. We will look at three special cases of this problem.

Problem (P1): There are N refuelings required for one aircraft to go from origin base, A, to destination base, B and the refueling points are considered to be instantaneous. The

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N refuelings will occur with one tanker from each of N refueling bases. A visual interpretation of this problem is presented in figure 1-1.

Problem (P2): There are N refuelings required for one aircraft to go from base A to base B, and the refueling occurs in such a way that it is not instantaneous, but requires time to offload from the tanker to the deployment aircraft. The N refuelings will occur with one tanker from each of N refueling bases. A visual interpretation of this problem is presented in figure 1-2.

Problem (P3): This is the same problem represented in Problem (P2) except we will now account for winds aloft in determining the optimal route.

These three problems successively build upon one another, as problem (P1) must be solved in order to solve problem (P2). Likewise, the algorithm for problem (P2) lays the foundation for the solution of problem (P3).

Literature Survey

The problems described above all involve finding a solution to a nonlinear minimization problem with constraints due to the fact that the optimization takes place on a sphere (an oblate spheroid to be exact). The
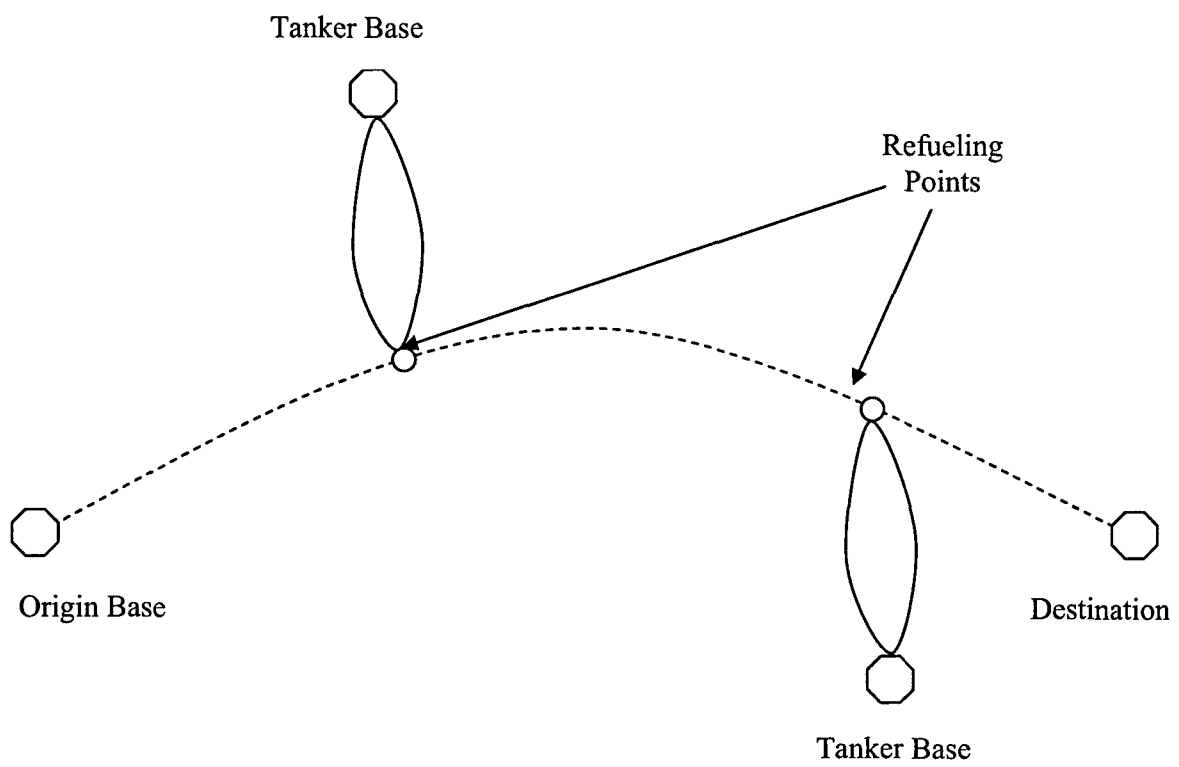
Tanker Base

Refueling
Points

Origin Base

Destination

Tanker Base

Figure 1-1. Graphical Representation of P1

Tanker Base

Refueling
Segments

Origin Base

Destination

Tanker Base
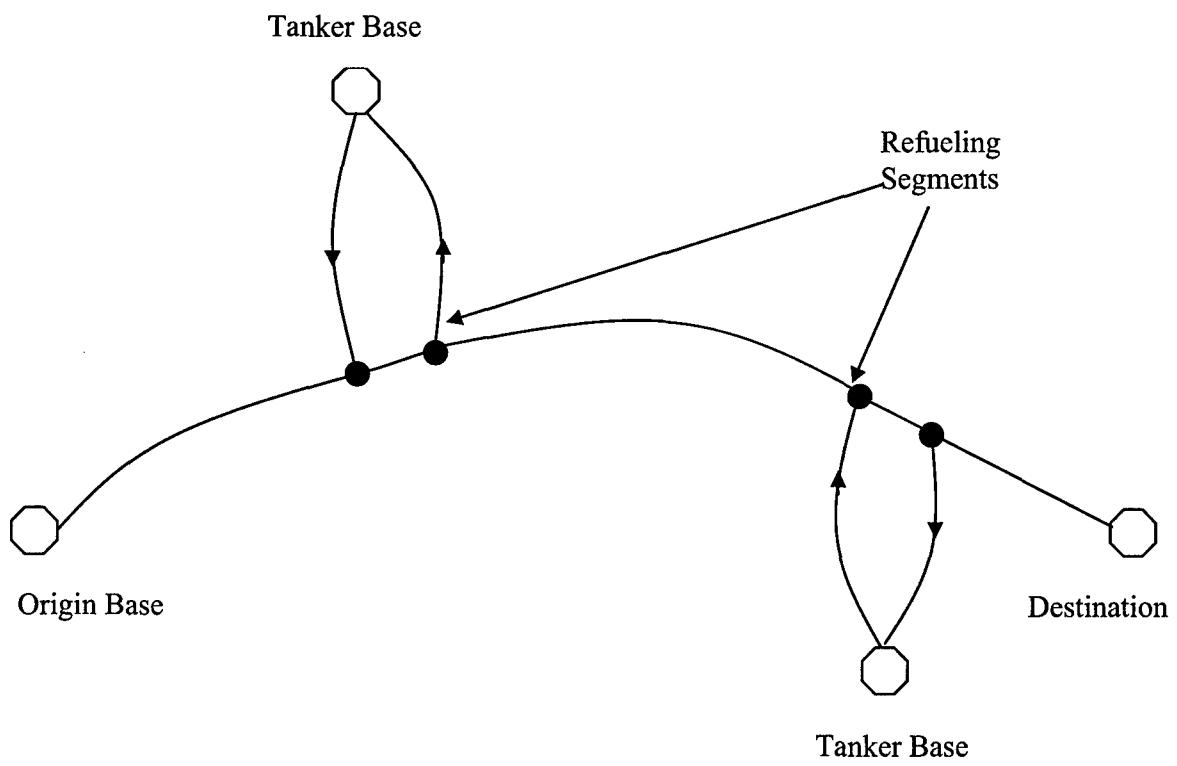
Figure 1-2. Graphical Representation of P2/P3

previously stated problems are all an extension of a problem solved by Yamani[11]. He solved the generic problem involving N aircraft deploying with one refueling required. This paper will use his solution to that problem and extend it to N refuelings and show that this problem also converges to an optimal solution. This problem, in its simplest form, can be characterized as a location problem on a sphere.

Yamani's work utilizes other significant results to achieve a solution. A similar problem is the spherical Weber problem with maximum distance constraints; however, that particular objective function is linear. Aly[1] showed that the search for an optimal solution to the spherical Weber problem, where demand points are not located entirely on a great circle, is contained within a convex hull of the demand points. Drezner[5] proved if the demand points lie on a great circle, the optimal solution also lies on this great circle. Katz and Cooper[7] showed the sufficient conditions under which a demand point is a local minimum.

Some work has also been accomplished on the Weber problem without constraints. Aly and Litwhiler[8] solved this problem using two different methods: the Map Projection Algorithm and Cyclic Search Algorithm. Ayken[2] derived the Cyclic Meridian Parallel Search and Geodesic Descent Algorithm to solve the Weber problem without constraints. All these algorithms

converge to local optima as long as the search is limited to a spherical radius of n/4. Additionally, it should be noted that the optimal point may not be unique.

Some researchers have looked at how to solve the problem with constraints. Love[9] developed a process that handled constraints through use of convexity and penalty functions. Ayken[2] found a solution to the Weber problem with distant constraints given that the solution is within a certain distance of previously known points.

Finally, Darnell and Loflin[3,4] used linear programming to solve a similar problem that utilized ground refueling, fixed intermediate stops, and a return to the origin. Waite[10] used stochastic dynamic programming to solve a similar problem. These problems had unrestricted but fixed refueling points with different fuel costs.

Assumptions

Some assumptions need to be made in order to start the solution process to these problems. These assumptions follow:

1. Aircraft will fly a great circle distance between any solution points found. The algorithm will use a version of the law of haversines to calculate this distance. This calculation is a very good first order calculation of the actual distance on the earth. This will be accomplished by the dists function in the

Matlog toolbox. This takes the earth's bulge into account when performing calculations. The haversine calculation follows[12]:

R = earth's radius (3,958.75 mi)
Δlat = lat2 - lat1
$\Delta long$ = long2 - long1
a = $\sin^2(\Delta lat/2)$ + cos(lat1) × cos(lat2) × $\sin^2(\Delta long/2)$
c = 2 × a tan 2($\sqrt{a}$, $\sqrt{(1-a)}$))
d = R × c

2.   Weather at altitude is assumed to be negligible except in Problem (P3).

3.   All deploying aircraft are of the same model. Similarly, the refueling aircraft are all assumed to be of the same model. In the particular algorithm being developed, the deploying aircraft is a C-5 and the refueling aircraft is a KC-10. If a requirement arises to analyze different types of aircraft beyond these, all that would be required is to change the fuel consumption functions introduced in chapter II.

4.   For the non-instantaneous problems (P2) and (P3), aircraft fuel consumption will be a constant during refueling segments due to the fact gross weight will be changing negatively due to fuel consumption AND also changing positively due to refueling.

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5. Takeoff and landing fuel consumption as well as position where these events occur will be input by the user and treated as constants.

6. Cargo weight must meet aircraft gross weight requirements also taking into account the fuel weight.

7. All aircraft will operate at a 99% maximum specific fuel rate, as this will ensure maximum speed while being only 1% less than the best fuel efficiency.

8. Aircraft are assumed to require a safety stock of fuel. This requirement affects range calculations, making the aircraft's effective range decrease. (due to the fact the safety stock fuel cannot be used for travel planning, but only for emergencies)

CHAPTER II

FUEL CONSUMPTION FUNCTION

Fuel Consumption Rate Function

In this chapter, we derive formulas for fuel consumption and aircraft range based on cargo weight (w_0), initial fuel (g_0 for deploying aircraft/h_0 for refueling aircraft), and flight altitude (alt). Yamani[11] derived these equations in the same manner with one notable difference. His equations depended solely on the gross weight of the aircraft. The following equations also incorporate flight altitude, as that was also deemed a significant factor in the model after performing the regression analysis.

The data is based on information in technical order 1C-5A-1-1[15] for the deploying aircraft and the KC-10A performance manual[14] for the refueling aircraft. Figure 2-1 shows an example specific range chart for the KC-10A at an altitude of 29,000 feet. These range charts exist for various altitudes. It is necessary to model the fuel consumption rate information contained in these charts by means of a mathematical formula.

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In order to do that, we will assume the aircraft operates at the 99% specific range since this maximizes velocity and is near optimal in fuel consumption.

Let

MPF(GW, Alt) =   the distance traveled in miles per 1,000 lbs of fuel burned given the aircraft gross weight, GW, and the flight altitude, Alt

Tables 2-1, 2-2, 2-3, 2-4, and 2-5 show fit, ANOVA, estimates, and effects tests based on the data from the C-5. A second degree polynomial model is used. We can see this gives an adjusted R^2 of over .99 which gives a sufficiently accurate representation of the data. Figure 2-2 shows a plot of the residuals of the data. We can see that the mean of the residuals is around zero and appear to be random which indicates a good model for our data. The KC-10 fuel consumption rate model also has an adjusted R^2 over .99 and is derived similarly. Figures 2-4 and 2-5 are plots of the actual versus predicted MPF(GW,Alt) against various altitudes and gross weights for the C-5 data. Notice in figure 2-4, the x-axis has four data points for every altitude. The four data points associated with each altitude are taken at progressively heavier gross weights. Thus, that is the reason you see the fuel consumption rate decreasing within the same

altitude. However, you will notice the general trend is an
increase in altitude corresponds to a decrease in fuel
consumption rate. In figure 2-5, you will notice the x-axis
has multiple repeating entries for different gross weights.
This is due to the fact that the data points taken at these
particular gross weights are taken at different altitudes.
The general trend is for the fuel consumption rate to decrease
as gross weight increases. The computed models of MPF(GW,
Alt) for the C-5 and KC-10 follow: (note that Alt^2 did not
turn out to be a significant factor for the C-5)

MPF$_{C-5}$(GW, Alt)

0.27678e-1+0.70599e-6 × Alt-0.42783e-7 × GW
-0.65146e-12 × Alt × GW+0.29811e-13 × GW^2

MPF$_{KC-10}$(GW, Alt)

$-0.2976658738e - 1 + 0.4268994944e - 5 \times Alt + 0.1481265454e - 7 \times GW$
$-0.4203287222e - 11 \times Alt \times GW - 0.3626383552e - 10 \times Alt^2$
$-0.6426861944e - 13 \times GW^2$
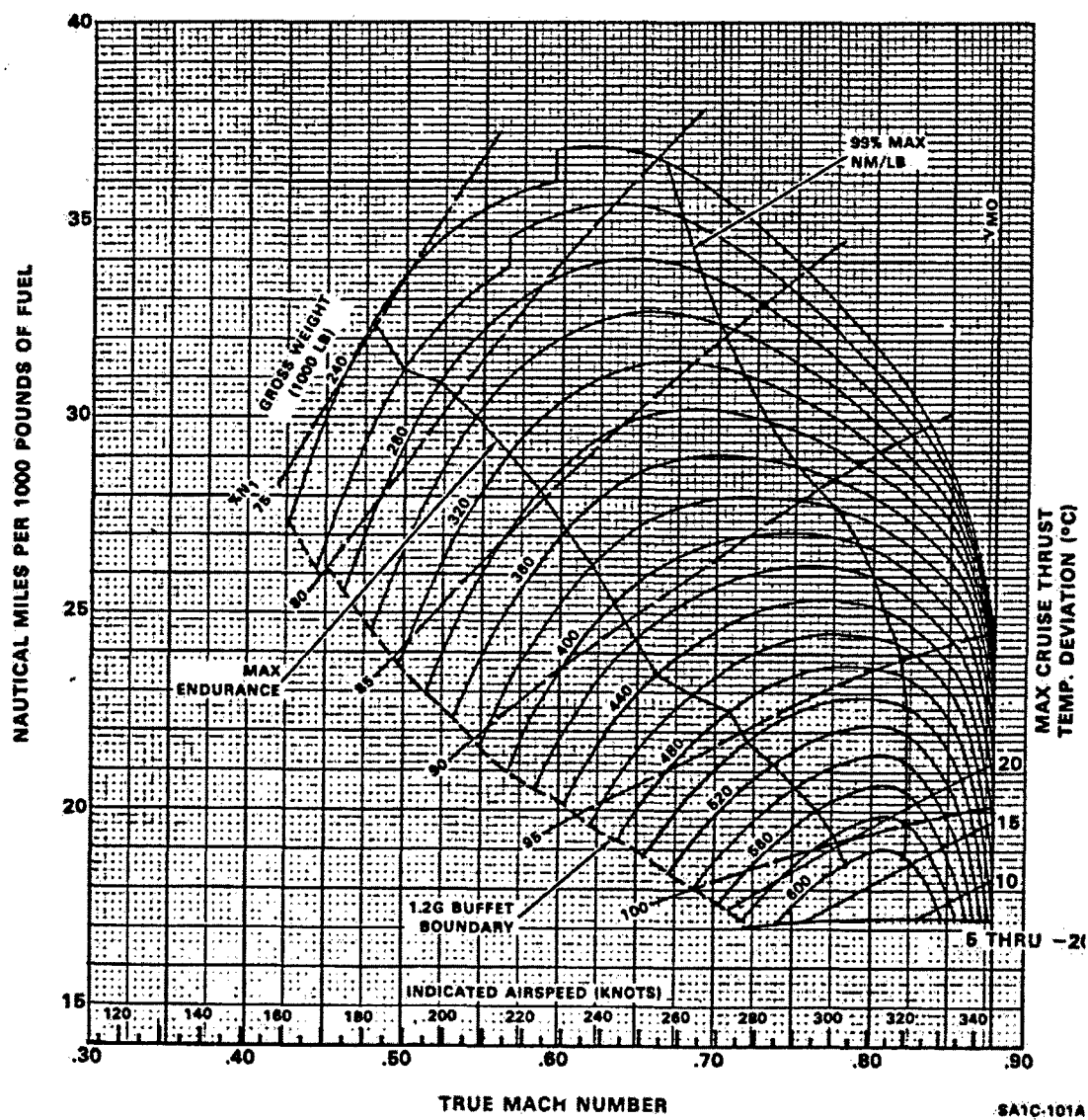
Figure 5-22. Three Engine Specific Range — 29,000 Feet

Figure 2-1. Sample Fuel Flow Chart

Table 2-1. C-5 Summary of Fit

RSquare	0.993319
RSquare Adj	0.992795
Root Mean Square Error	0.000555
Mean of Response	0.022731
Observations (or Sum Wgts)	56

Table 2-2 C-5 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	0.00233780	0.000584	1895.56
Error	51	0.00001572	3.083e-7	Prob > F
C. Total	55	0.00235352		<.0001

Table 2-3 C-5 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0273714	0.000369	74.19	<.0001
Alt	3.5885e-7	6.63e-9	54.12	<.0001
GW	-2.632e-8	4.98e-10	-52.83	<.0001
(Alt-23500)*(GW-532857)	-6.51e-13	4.33e-14	-15.03	<.0001
$(GW-532857)^2$	2.981e-14	3.66e-15	8.14	<.0001

Table 2-4 C-5 Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Alt	1	1	0.00090313	2929.133	<.0001
GW	1	1	0.00086051	2790.931	<.0001
Alt*GW	1	1	0.00006969	226.0199	<.0001
GW^2	1	1	0.00002042	66.2187	<.0001

Figure 2-2 (Residual by Predicted Plot)
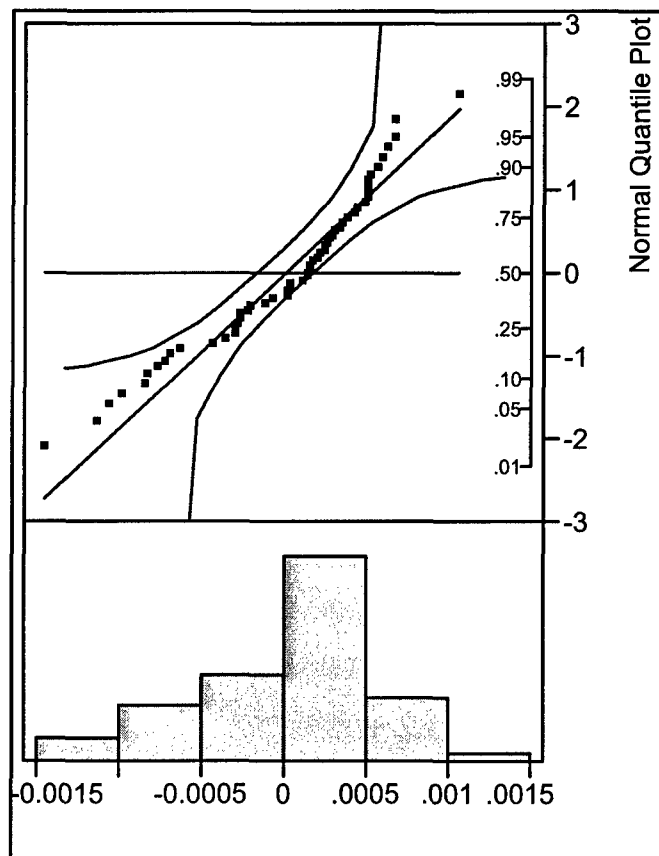
Figure 2-3 (Normal Probability Plot of Residuals)

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C-5 Actual Vs Predicted MPF
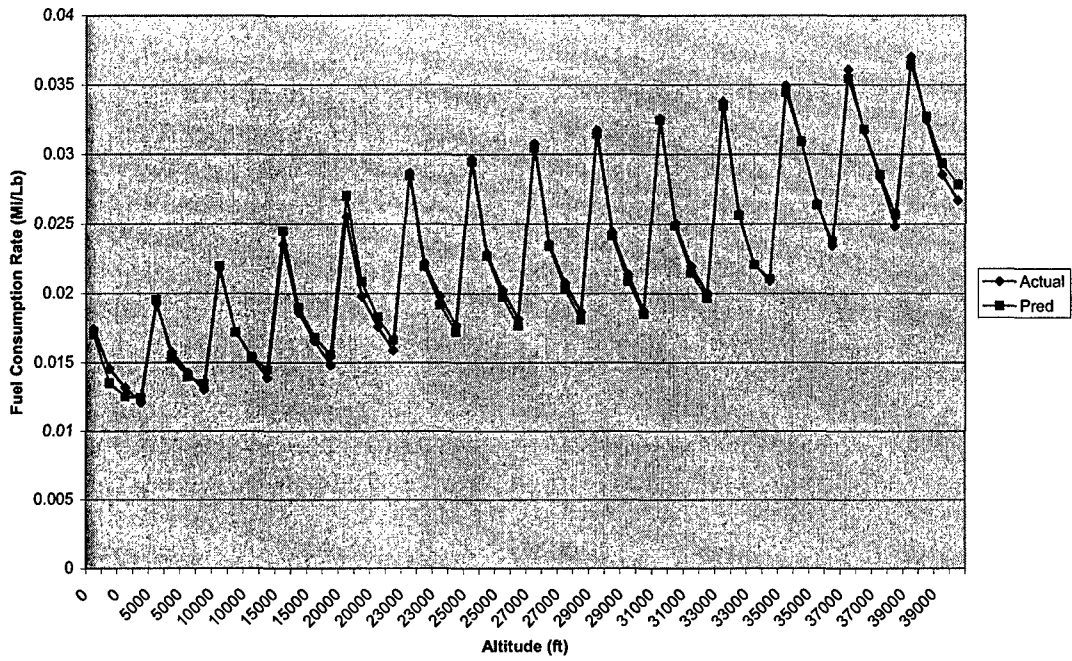
Figure 2-4. C-5 Actual vs Predicted (Altitude)

C-5 Actual Vs Predicted
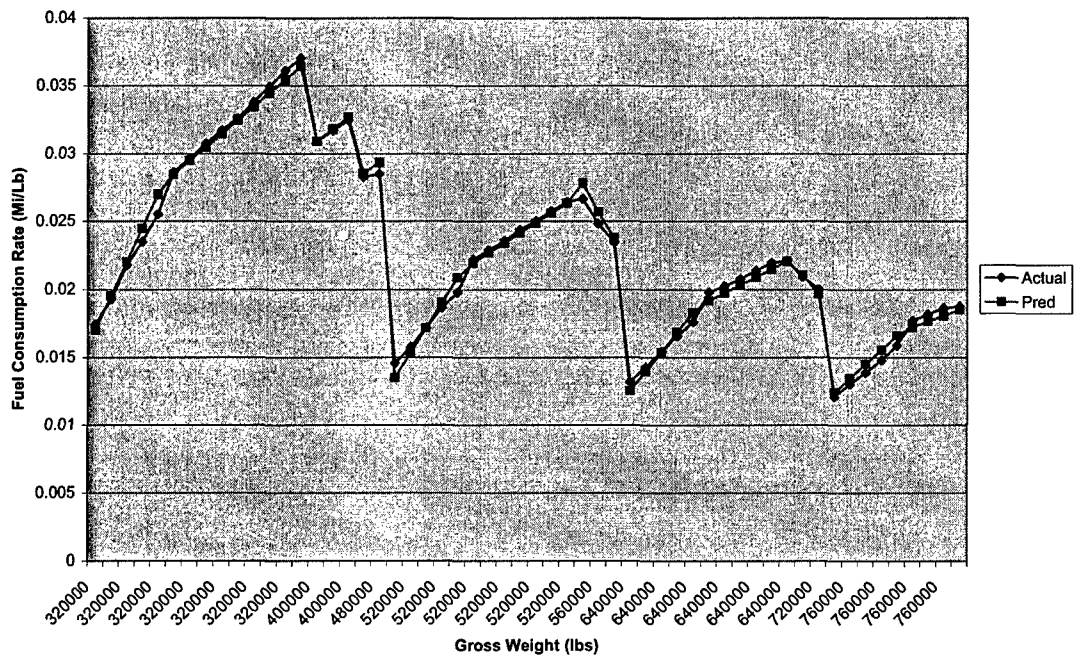
Figure 2-5. C-5 Actual vs Predicted (Gross Weight)

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Tables 2-5, 2-6, 2-7, and 2-8 show the regression analysis for the KC-10 fuel flow function.

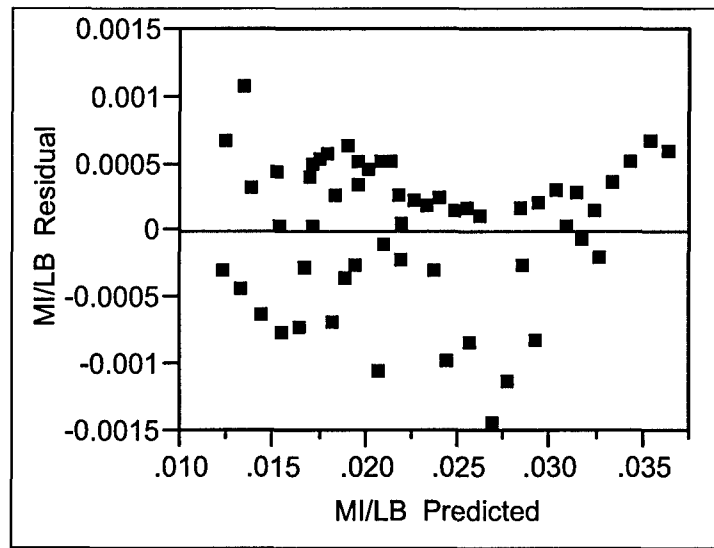Table 2-5. KC-10 Summary of Fit

RSquare	0.997985
RSquare Adj	0.997527
Root Mean Square Error	0.000431
Mean of Response	0.036367
Observations (or Sum Wgts)	28

Table 2-6 KC-10 Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	0.00202623	0.000405	2179.398
Error	22	0.00000409	1.859e-7	Prob > F
C. Total	27	0.00203033		<.0001

Table 2-7 KC-10 Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.059863	0.001224	48.90	<.0001
Alt	1.9332e-7	2.741e-8	7.05	<.0001
GW	-8.529e-8	1.034e-9	-82.51	<.0001
(Alt-35,000)*(GW-365,714)	-4.2e-12	3.17e-13	-13.24	<.0001
$(Alt-35,000)^2$	-3.63e-11	6.49e-12	-5.59	<.0001
$(GW-365,714)^2$	6.427e-14	8.98e-15	7.15	<.0001

Table 2-8 KC-10 Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Alt	1	1	0.00000925	49.7555	<.0001
GW	1	1	0.00126590	6807.964	<.0001
Alt*GW	1	1	0.00003260	175.2967	<.0001
Alt^2	1	1	0.00000581	31.2693	<.0001
GW^2	1	1	0.00000952	51.1864	<.0001

Aircraft Range Function

It is important to know the distance, d, an aircraft can fly given its initial fuel (g_0), flight altitude (alt), and cargo weight (w_0). First, let us consider the initial gross weight (GW) of the aircraft.

$GW = EW + g_0 + w_0$ (note that this formulation only holds

true if the aircraft has not used any fuel yet)

where EW is the empty weight of the aircraft. During flight,

$GW = EW + f + w_0$

where f is the amount of fuel left. One can easily see that EW and w_0 do not change. Therefore,

$\partial GW = \partial f$

Let

$R_j(g_0, w_0, alt) =$ the range of aircraft j given an initial

fuel, cargo weight, and flight altitude

Now, we can surmise that an optimal solution will occur only if the deploying aircraft finishes with little or no fuel. Therefore, the ending gross weight is,

$Final\ GW = EW + w_0$ (as $f = 0$)

We can now integrate the $MPF(GW, Alt)$ function we found earlier for each aircraft and find its range function:

$$R(g_0, w_0, alt) = \int_{EW+w_0}^{EW+w_0+g_0} MPF(GW)dGW$$

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$$= \int_0^{g_0} MPF(EW + w_0 + f)df$$

We get the following:

$R_{C-5}(g_0, w_0, alt)$:

$0.9937160432e - 14 \times g_0^3 - 0.2139143481e - 7 \times g_0^2$

$-0.3257322132e - 12 \times g_0^2 \times Alt + 0.2981148130e - 13 \times g_0^2 \times EW$

$+0.2981148130e - 13 \times g_0^2 \times w_0 + 0.2767819883e - 1 \times g_0$

$+0.7059890563e - 6 \times Alt \times g_0 - 0.4278286962e - 7 \times EW \times g_0$

$-0.4278286962e - 7 \times w_0 \times g_0 - 0.6514644265e - 12 \times g_0 \times Alt \times EW$

$-0.6514644265e - 12 \times g_0 \times Alt \times w_0 + 0.2981148130e - 13 \times g_0 \times EW^2$

$+0.5962296260e - 13 \times g_0 \times EW \times w_0 + 0.2981148130e - 13 \times g_0 \times w_0^2$

$R_{KC-10}(h_0, y_0, alt)$:

$0.2142287315e - 13 \times h_0^3 - 0.2101643611e - 11 \times h_0^2 \times Alt$

$+0.7406327270e - 8 \times h_0^2 + 0.6426861945e - 13 \times h_0^2 \times EW$

$+0.6426861945e - 13 \times h_0^2 \times y_0 - 0.2976658738e - 1 \times h_0$

$+0.4268994944e - 5 \times Alt \times h_0 + 0.1481265453e - 7 \times EW \times h_0$

$+0.1481265453e - 7 \times y_0 \times h_0 + 0.6426861944e - 13 \times h_0 \times EW^2$

$+0.1285372389e - 12 \times h_0 \times EW \times y_0 + 0.6426861944e - 13 \times h_0 \times y_0^2$

$-0.4203287222e - 11 \times h_0 \times Alt \times EW - 0.4203287222e - 11 \times h_0 \times Alt \times y_0$

$-0.3626383552e - 10 \times h_0 \times Alt^2$

Fuel Consumption Function

It is also helpful to know $F_j(g_0, w_0, alt, d)$, the amount of fuel consumed when an aircraft flies a given distance, d, with an initial fuel, g_0/h_0, flight altitude, alt, and cargo weight w_0/y_0. In order to find this value, we must also know how much fuel is left in the aircraft after completion of the leg. We will denote this value as g_1/h_1. Now,

$F_j(g_0, w_0, alt, d) = g_0 - g_1$

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Since we know g_0, we must only find g_1 in order to find F_j (g_0, w_0, alt, d). We do this through the following equation,

$$d = \int_{g_1}^{g_0} MPF(EW + w_0 + f)df$$

Doing this, we get the following:

$F_{c-5}(g_0, w_0, alt, d)$:

$g_0 + (-141131967 \times Alt \times w_0 + 0.3950720052e19$

$-141131967 \times Alt \times EW - 0.2053949494e13 \times EW$

$-0.2053949494e13 \times w_0 + 0.1449491222e15 \times Alt$

$+sqrt(0.8215797968e33 \times d + 0.5797558638e21 \times Alt \times g_0^2$

$+0.8437417038e25 \times EW \times g_0 + 0.8437417038e25 \times w_0 \times g_0$

$+0.1145306807e34 \times Alt + 0.1159511729e22 \times Alt \times w_0 \times EW$

$+0.3983646422e17 \times Alt^2 \times EW \times g_0 + 0.2101024802e29 \times Alt^2$

$+0.5645278680e29 \times Alt \times d + 0.1991823211e17 \times Alt^2 \times g_0^2$

$-0.4091390946e23 \times Alt^2 \times g_0 - 0.1622915890e32 \times EW$

$-0.1622915890e32 \times w_0 + 0.1560818893e38$

$-0.1622915889e32 \times g_0 + 0.4218708516e25 \times g_0^2$

$+0.1159511728e22 \times Alt \times EW \times g_0 +$

$0.1159511728e22 \times Alt \times w_0 \times g_0 +$

$0.3983646422e17 \times Alt^2 \times w_0 \times EW$

$-0.1710582136e28 \times Alt \times g_0 - 0.1710582136e28 \times Alt \times EW$

$-0.1710582136e28 \times Alt \times w_0 + 0.3983646422e17 \times Alt^2 \times w_0 \times g_0$

$+0.1991823211e17 \times Alt^2 \times w_0^2 + 0.5797558643e21 \times Alt \times w_0^2$

$-0.4091390946e23 \times Alt^2 \times w_0 + 0.1991823211e17 \times Alt^2 \times EW^2$

$+0.5797558643e21 \times Alt \times EW^2 - 0.4091390946e23 \times Alt^2 \times EW$

$+0.4218708522e25 \times EW^2 + 0.8437417044e25 \times EW \times w_0$

$+0.4218708522e25 \times w_0^2)) \div (0.2053949492e13 + 141131967 \times Alt)$

20

$F_{KC-10}(h_0, y_0, alt, d):$

$h_0 + (0.8539003945e16 - 0.1499862837e14 \times Alt$

$-0.6936177892e12 \times EW - 0.6936177892e12 \times y_0$

$+38507457.01 \times Alt \times EW + 38507457.01 \times Alt \times y_0$

$-0.7812500000e - 2 \times sqrt(-0.1750430987e25 \times EW \times Alt \times h_0$

$-0.1750430987e25 \times y_0 \times Alt \times h_0 - 0.4196707428e34 \times Alt$

$+0.1576486953e29 \times EW \times h_0 + 0.1576486953e29 \times y_0 \times h_0$

$+0.3516704614e30 \times Alt \times h_0 + 0.3516704613e30 \times Alt \times EW$

$+0.3516704613e30 \times Alt \times y_0 - 0.1775661541e36 \times d$

$-0.1940784755e33 \times EW - 0.1940784755e33 \times y_0$

$-0.1940784756e33 \times h_0 + 0.7882434769e28 \times h_0^2$

$-0.8752154935e24 \times Alt \times h_0^2 + 0.1194632616e37$

$+0.4858918487e20 \times Alt^2 \times EW \times y_0 - 0.1750430987e25 \times EW \times Alt \times y_0$

$+0.4858918486e20 \times y_0 \times Alt^2 \times h_0 + 0.4858918486e20 \times EW \times Alt^2 \times h_0$

$+0.3685725846e31 \times Alt^2 - 0.1892545453e26 \times Alt^2 \times EW$

$-0.1892545453e26 \times Alt^2 \times y_0 + 0.7882434765e28 \times EW^2$

$+0.1576486953e29 \times EW \times y_0 - 0.8752154935e24 \times Alt \times EW^2$

$+0.7882434765e28 \times y_0^2 - 0.8752154935e24 \times Alt \times y_0^2$

$+0.2429459243e20 \times Alt^2 \times EW^2 + 0.2429459243e20 \times Alt^2 \times y_0^2$

$+0.2429459242e20 \times Alt^2 \times h_0^2 + 0.9857908992e31 \times Alt \times d$

$-0.1892545452e26 \times Alt^2 \times h_0)) \div (-0.6936177894e12 + 38507457 \times Alt)$

The above equations are based on the quadratic model with no two way interactions considered of the fuel consumption rate equation. The adjusted R^2 was about .989 and .985 for the KC-10 and C-5 models, respectively.

From the research performed by Yamani, we know the fuel consumption function is a concave and increasing function of the distance[11]. Additionally, the fuel consumption function is a convex and increasing function of w_0[11].

Fuel Requirement Function

The last function of interest is the fuel requirement function, $FR_j(w_0, alt, d)$. This function represents the exact amount of fuel required to fly a distance, d, at flight altitude, alt, and cargo weight, w_0. We know the initial amount of fuel, g_0/h_0 must be greater than or equal to the fuel required (otherwise the aircraft would run out of fuel). So, set

$$R_j(g_0, w_0, alt) = d$$

and solve for g_0 to get the fuel required. We get:

$FR_{C-5}(w_0, alt, d):$

$$
\begin{aligned}
(&-0.2053949494e13 \times w0 + 0.3950720052e19 \\
&+0.1449491222e15 \times Alt - 0.2053949494e13 \times EW \\
&-141131967.0 \times Alt \times EW - 141131967.0 \times Alt \times w_0 \\
&+sqrt(0.1145306807e34 \times Alt - 0.1622915890e32 \times EW \\
&-0.1622915890e32 \times w_0 + 0.4218708522e25 \times w_0^2 \\
&+0.8437417044e25 \times w_0 \times EW + 0.5797558643e21 \times Alt \times w_0^2 \\
&+0.2101024802e29 \times Alt^2 - 0.4091390946e23 \times Alt^2 \times EW \\
&-0.4091390946e23 \times Alt^2 \times w_0 + 0.4218708522e25 \times EW^2 \\
&+0.5797558643e21 \times Alt \times EW^2 + 0.1991823211e17 \times Alt^2 \times EW^2 \\
&+0.1991823211e17 \times Alt^2 \times w_0^2 + 0.3983646422e17 \times Alt^2 \times EW \times w_0 \\
&+0.1159511729e22 \times w_0 \times Alt \times EW + 0.1560818893e38 \\
&-0.1710582136e28 \times Alt \times EW - 0.1710582136e28 \times Alt \times w_0 \\
&-0.8215797968e33 \times d \\
&-0.5645278680e29 \times d \times Alt)) \div (0.2053949492e13 + 141131967 \times Alt)
\end{aligned}
$$

$FR_{KC-10}(y_0, alt, d)$:

$$
\begin{aligned}
(&-0.8878307702e14 \times y_0 + 0.1092992505e19 - 0.1919824431e16 \times Alt \\
&-0.8878307702e14 \times EW + 4928954497 \times Alt \times EW \\
&+4928954497 \times Alt \times y_0 - xsqrt(0.1576486953e29 \times y_0 \times EW \\
&-0.8752154934e24 \times Alt \times y_0^2 + 0.3685725846e31 \times Alt^2 \\
&-0.1892545453e26 \times Alt^2 \times EW - 0.1892545453e26 \times Alt^2 \times y_0 \\
&+0.7882434764e28 \times EW^2 - 0.8752154934e24 \times Alt \times EW^2 \\
&+0.2429459243e20 \times Alt^2 \times EW^2 + 0.2429459243e20 * Alt^2 \times y_0^2 \\
&+0.1194632616e37 + 0.4858918487e20 \times Alt^2 \times EW \times y_0 \\
&-0.1750430987e25 \times y_0 \times Alt \times EW - 0.4196707428e34 \times Alt \\
&+0.3516704613e30 \times Alt \times y_0 + 0.3516704613e30 \times Alt \times EW \\
&-0.1940784755e33 \times EW - 0.1940784755e33 \times y_0 + 0.7882434764e28 \times y_0^2 \\
&+0.1775661541e36 \times d - 0.9857908994e31 \times d \times Alt)) \div \\
(&-0.8878307703e14 + 4928954497 \times Alt)
\end{aligned}
$$

CHAPTER III

PROBLEM (P1)—MODEL AND SOLUTION

---

1 Aircraft

N Aerial Refuelings

(N+1 legs)

Instantaneous Refueling

Problem Description

This problem is an extension of a problem solved by Yamani[11]. He considered and solved the problem for one aerial refueling with the assumption that refueling is instantaneous. Consider the case where there is one aircraft that is to be deployed from base A to base B. However, the distance, d, from base A to base B is greater than the range, $R_{deploy}(g_0, w_0, alt)$, of the deploying aircraft. Therefore, 1 or more aerial refuelings must be accomplished. The tanker aircraft would come from predetermined air bases and refuel the deploying aircraft in midair enroute to base B. At the end of each refueling, the tanker aircraft would return to its origin base. The objective in this case would be to determine the N

refueling points that minimize total fuel consumption of both the deploying aircraft and the N tanker aircraft used. Additionally, we shall determine how much fuel is added to the deploying aircraft at each refueling point. Let

D_i = the distance of leg i of the deployment (for example, the distance from the origin base to the first refueling point would be D_1 and the distance from refueling point 1 to refueling point 2 would be D_2, etc.)

T_i = the distance from aerial refueling base i to aerial refueling point i

$R_j(g_0, w_0, alt)$ = the range of aircraft j given its initial fuel, cargo weight, and planned flight altitude (j = tanker or deploying aircraft)

$F_{max}(w_0, j)$ = the maximum fuel aircraft j can carry given its cargo weight (j = tanker or deploying aircraft)

$W_{max}(j)$ = the maximum gross weight for aircraft j (j = tanker or deploying aircraft)

$F_j(g_0, w_0, alt, d)$ = amount of fuel used to fly a distance d given an initial fuel amount, cargo weight, w_0, and altitude by aircraft j (j = tanker or deploying aircraft)

$f_i =$ amount of fuel consumed by the deploying aircraft on leg i

$$= F_{deploy}\left(g_0 + \sum_{j=1}^{i}(x_{j-1} - f_{j-1}), w_0, D_i, alt\right)$$

$x_i =$ amount of fuel transferred from the tanker to the deploying aircraft at refueling point i

$r_i =$ amount of fuel used by tanker aircraft i to fly from its home base to the refueling point

$$= FR_{tanker}(h_0, y_0, T_i, alt)$$

$s_i =$ amount of fuel used by tanker aircraft i to fly from its refueling point back to its home base

$$= F_{tanker}(h_0 - x_i - r_i, y_0, T_i, alt)$$

$w_0 =$ cargo weight of deploying aircraft

$y_{0i} =$ cargo weight of tanker aircraft i

$g_0 =$ initial fuel of deploying aircraft

$h_{0i} =$ initial fuel of tanker aircraft i

Several constraints must be satisfied in determining the N optimal refueling points. First, the distance of each leg cannot exceed the maximum range of the deploying aircraft.

$$D_i \leq R_{deploy}\left(g_0 + \sum_{j=1}^{i}(x_{j-1} - f_{j-1}), w_0, alt\right)$$

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Second, the tanker aircraft must be able to reach the
refueling point and then have enough fuel to return back to
its original base. This algorithm will assume that getting to
the aerial refueling point is not a problem for the tanker,
however, the algorithm checks to make sure that there the
aerial refueling aircraft has enough range to make the trip
back to its home base from the aerial refueling point.

$$T_i \leq R_{tan\ ker}(h_{0i} - x_i - F_{tanker}(h_{0i}, y_{0i}, T_i), y_{0i}, alt)$$

Similarly, the amount of fuel consumed by both the tanker and
deploying aircraft for each leg must be less than either its
maximum fuel given its cargo weight, or for leg 1, less than
its initial fuel, g_0 or h_0:

$$F_{deploy}(g_0, w_0, alt, D_1, alt) \leq g_0$$

$$F_{deploy}(g_0 + \sum_{j=1}^{i}(x_{j-1} - f_{j-1}), w_0, D_i, alt) \leq g_0 + \sum_{j=1}^{i} x_{j-1} - f_{j-1}$$

$$F_{tan\ ker}(h_{0i}, y_0, T_i, alt) \leq h_{0i}$$

$$F_{tan\ ker}(h_{0i} - x_i - r_i, y_0, T_i, alt) \leq h_{0i} - x_i - r_i$$

Additionally, the aircraft must meet its weight requirements
at each leg, i. Thus:

$$W_{max}(deploy) \geq g_0 + w_0 + \sum_{j=1}^{i}(x_{j-1} - f_{j-1})$$

$$W_{max}(tanker) \geq h_{0i} + y_{0i}$$

Finally, it should be noted that the tanker aircraft must also be able to offload the necessary fuel to the deploying aircraft, while also being able to reach its final destination.

Formulation of (P1)

The problem can be stated mathematically as follows: find the N coordinates (latitude, longitude) and amount of fuel transferred from the tanker to the deploying aircraft at each refueling point that will

$$(\text{P1}) \quad \text{Minimize} \quad \sum_{i=1}^{N} f_i + r_i + s_i$$

s.t.

$$F_{deploy}(g_0 + \sum_{j=1}^{i}(x_{j-1} - f_{j-1}), w_0, D_i, alt) \le g_0 + \sum_{j=1}^{i} x_{j-1} - f_{j-1}$$

$$D_i \le R_{deploy}(g_0 + \sum_{j=1}^{i}(x_{j-1} - f_{j-1}), w_0, alt)$$

$$F_{tan\,ker}(h_{0i}, y_0, T_i, alt) \le h_{0i}$$

$$F_{tan\,ker}(h_{0i} - x_i - r_i, y_0, T_i, alt) \le h_{0i} - x_i - r_i$$

$$T_i \le R_{tan\,ker}(h_{0i}, y_{0i}, alt)$$

$$T_i \le R_{tan\,ker}(h_{0i} - x_i - F_{tan\,ker}(h_{0i}, y_{0i}, T_i), y_{0i}, alt)$$

$$W_{max}(deploy) \ge g_0 + w_0 + \sum_{j=1}^{i}(x_{j-1} - f_{j-1})$$

$$W_{max}(tanker) \ge h_{0i} + y_{0i}$$

Note: the tankers are all assumed to be the same model, and thus, have the same weight characteristics and limitations.

Solution Procedure for Problem (P1)

1.   Pick N initial feasible points. (discussion of how to pick these points is at the end of the chapter)

2.   Calculate the fuel cost involved with using these N initial feasible points.

3.   Set an N length vector, λ (step length), a minimum λ, a tolerance (*TOL*), and let *i=1*.

4.   Fix all N points except for point *i*. Create four 'new' points by moving point *i* north, south, west, and east by λ_i.

> a. Calculate the fuel cost of each of the four 'new' points and check feasibility.
>
> > 1.   If the 'new' point is not feasible, the fuel cost associated with that point is infinity.
>
> b. Compare the smallest fuel cost of the four 'new' points to the 'old' fuel cost. If 'new' fuel cost < fuel cost, set point i to the point corresponding to the 'new' fuel cost and fuel cost is now equal to 'new' fuel cost. Set difference = fuel cost – 'new' fuel cost.
>
> > 1.   Else, set $\lambda_i = \lambda_i/2$.

5.   Check the vector λ against the minimum λ requirement and difference against the user defined tolerance

movement. If *difference < TOL* & max *(λ) < (user defined min λ)*, the algorithm is complete. If not, continue.

6. Increment *i* unless *i=N*, then set *i=1* again.

7. Go to step 4.

Yamani[11], using the fact that the distance from a given point r is an s-convex function within a circle of radius Π/2 and center r (Drezner and Weslowsky[6]) and the search for an optimal solution to the spherical Weber problem can be restricted to the spherical convex hull of the demand points (Aly[1]), showed that we can limit our search for refueling points to the convex hull of the endpoints of each leg i, along with the associated aerial refueling base point. Yamani[11] also showed that the objective function is s-convex. The multiple aerial refueling problem is clearly just an extension of the single refueling station problem. We can apply the same procedure if we temporarily fix the points before and after the refueling point of interest. Since the objective function is s-convex, we know that if we move in a direction that improves the objective function, we are moving toward the optimal position. When it is no longer possible to improve the objective function in any direction and/or step length, optimality is reached.

Obtaining an Initial Feasible Solution

1. Let $j=1$ and start with the origin base, (x_{j-1}, y_{j-1}) and first aerial refueling base, (ARx_j, ARy_j).

2. Let the jth refueling point be $(z_j, w_j) = ((x_{j-1}+ARx_j)/2, (y_{j-1}+ARy_j)/2)$ for all j initially. (Note that since this is on a sphere, these interpolated points are not midpoints. This does not matter for our purposes here.)

3. Check feasibility of the new found point.

   a. If point j is too far from the aerial refueling base (range of tanker, $R_{tanker}(h_0, y_0, alt)$, is less than the distance from the aerial refueling base to the refueling point), move point j incrementally closer to the aerial refueling base; $((9*z_j + ARx_j)/10, (9*w_j + ARy_j)/10)$.

   b. If point j is out of range of the deploying aircraft $(R_{deploy}(g_0, w_0, alt))$, move point j incrementally closer to the previous refueling point or origin; $((x_{j-1}+9*z_j)/10, (y_{j-1} + 9*w_j)/10)$.

   c. If point j is out of range of the next refueling point (or destination if $j=N$), move point j closer to the next refueling point; $((x_{j+1}+9*z_j)/10, (y_{j+1} + 9*w_j)/10)$.

4. If all points are feasible, you are complete. If not, let $j=j+1$ or if $j=N$, set $j=1$.

Convergence Proof for N>1 Aerial Refuelings

We need to show that at each and every successive iteration, the total fuel consumption for both deployment and refueling aircraft is less than or equal to the previously calculated total fuel consumption. Yamani showed this is true for $N=1$. Assume that we know the optimal solution of refueling points, call this set S and it takes k iterations to achieve. Now, also assume that at some iteration, $i<k$, the calculated total fuel consumption increases (diverges). Remember, the algorithm moves one refueling point at a time, and fixes all other points. Let S' be the solution set associated with the $i-1$ iteration. According to the above algorithm, if that were the case, we would not change S' due to the fuel cost increasing, and we would divide λ_j for point j by 2 and perform the next iteration. Assume the total fuel consumption keeps diverging for point j. Eventually λ_j would be sufficiently close to zero and the 'new' points created by moving a distance λ_j would be the same as the point in the current solution set S'. Then $S'(j)=S(j)$ and this point is in the optimal solution set for point j. This same concept can be shown to be true for all points of j from 1 to N. Therefore, the objective function never increases from one iteration to the next, and if it reaches a point where it never again decreases, we have found the optimum. The worst

that could happen is the objective function maintains the same value. Therefore, since Yamani[11] showed the fuel consumption functions are s-convex, we know via Drezner and Wesolowsky[6] that the procedure converges to a global optimum solution.

CHAPTER IV

PROBLEM (P2)—MODEL AND SOLUTION

---

1 Aircraft

N Aerial Refuelings

(N+1 legs)

Non-Instantaneous Refueling

Problem Description

This problem is the exact same one described in the last chapter except for the fact that refueling is assumed to consist of a start point and end point, instead of being an instantaneous procedure. Therefore, this will mean that a deployment will consist of *2N+1* legs. This chapter will introduce a heuristic that will obtain a 'near' optimal solution to the problem.

The mathematical formulation and solution procedure (at least the start of it) of this problem is the same as Problem (P1).

Solution Procedure for Problem (P2)

1. Solve Problem (P2) using Problem (P1) method. This gives
    you N instantaneous optimal refueling points. We will
    call these (x_i, y_i) $i=1..N$. Let $i=0$.

2. Calculate the fuel remaining in the object aircraft at
    point i using $F_j(g_0, w_0, alt, d)$.

3. Calculate $FR_{deploy}(w_0, alt, D_{i+1})$. Once we have (2) and (3),
    we know how much fuel we must offload from the tanker,
    x_{i+1}, in order to make the next refueling point by
    calculating how much fuel we have left at point i,
    $F_{deploy}(g_0, w_0, alt, D_i)$, and also using $FR_{deploy}(w_0, alt,$
    $D_{i+1})$, the fuel required to go from point i to point $i+1$.
    The user will provide the program with an offload rate, α,
    and a speed, β, can be calculated using regression based
    on aircraft altitude and weight.

4. For the purposes of the refueling legs, fuel consumption,
    γ, will be assumed to be constant on just those legs.
    This is due to the fact that the fuel consumption
    functions will not accurately predict fuel usage when the
    weight of the aircraft is increasing in the middle of
    flight (due to aerial refueling).

5.   Once we have obtained (4) and also have a constant fuel
    consumption rate, γ, we can calculate the refueling

distance leg length. For each point i, we have a refueling distance,

$$q_i = FR(w_0, D_{i+1}, alt) \div \alpha$$

Note: it is imperative when making these calculations that you make sure your units match.

6. Next, we calculate 10 different refueling segments for segment i and pick the best one in terms of fuel consumption. (Note: the number of refueling segments may change as I continue performing sensitivity analyses to determine the number that maximizes performance and efficiency of the algorithm.)

   a. Bearing Calculation from point $i+1$ to point i:

   $$\nabla lon = lon(i + 1) - lon(i)$$
   $$\nabla lat = lat(i + 1) - lat(i)$$
   $$Br = a \tan 2(\sin(\nabla lon) \times \cos(lat(i + 1),$$
   $$\cos(lat(i) \times \sin(lat(i + 1)$$
   $$- \sin(lat(i) \times \cos(lat(i + 1) \times \cos(\nabla lon))$$

   b. Calculation of a point given a bearing, Br, and distance, d, from another point (use the instantaneous refueling point):

   $$lat1 = a \sin(\sin(lat) * \cos(d) + c \, os(lat) * \sin(d) * \cos(Br));$$
   $$lon1 = lon + a \tan 2((\sin(Br) * \sin(d) * \cos(lat)),$$
   $$\cos(d) - (\sin(lat) * \sin(lat1)));$$

   This point is the beginning of the refueling segment. Calculate this point for $k_i = (q_i/10) * p$ where $p = 1$ to 10.

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7. Since we have found the beginning refueling point, we must now find the end refueling point for each refueling segment. Let A be the instantaneous refueling point (x_{ij}, y_{ij}). Let B be the next instantaneous refueling point (x_{i+1}, y_{i+1}). Let D be the beginning refueling point, (a,b). Now, given a great circle route AB and a point D, we can find the points on AB that lie a distance q_i from D. This will give us the ending refueling point[13].

    a. $M=bearing(AD)-bearing(AB)$

    $L=dist(AD)$

    $$r=\sqrt{\cos^2(L) + \sin^2(L) * \cos^2(M)}$$

    $p = atan2(sin(L)*cos(M),cos(L))$

    $dp = p +- acos(cos(q_i)/r)$

    Repeat this for all $D = k_i$ and call this solution point kk_i.

8. Now, check all 10 possible solutions for beginning and ending refueling point i, and pick the segment that minimizes fuel consumption and also meets range function feasibility for both tanker and deployment aircraft. Increment i and return to step 2. If $i=N$, go to step 10.

9. Set $i=1$. In the same way as Problem (P1), move the beginning and ending refueling points for each i a fixed step length, λ_i, to the north, east, south, and west.

10. If we can move the beginning and ending refuel points and achieve a smaller fuel cost, the associated points are the new beginning and ending refuel points for step i. Otherwise, $\lambda_i = \lambda_i/2$.

11. Increment i or if $i=N$, set $i=1$ and goto step 10. Stop the procedure when all λ_i are sufficiently small AND the difference between fuel costs are negligible.

12. Repeat steps 6-8 again, checking if the angle of the refueling segment improves fuel performance.

CHAPTER V

PROBLEM (P3)-MODEL AND SOLUTION

---

1 Aircraft

N Aerial Refuelings

Non-Instantaneous Refueling

Winds Aloft Factored into Fuel Cost Calculations

Problem Description

Suppose we have an aircraft that needs to be deployed from base A to base B and the distance between the two bases is greater than the range of the aircraft. Thus, 1 or more aerial refuelings are required. There are predetermined refueling bases from which refueling aircraft takeoff and land to support these deploying aircraft en route to the destination. In the real world, we must also account for winds when flying our aircraft. A significant headwind causes an aircraft to use more fuel than no wind or even, a tailwind. The problem then is: where do the optimal refueling points occur when correcting for forecast winds?

Solution Procedure

The procedure mimics the problem (P2) solution with one
notable exception: the actual calculation of the fuel cost.
Everything else remains the same.

Calculation of Fuel Cost

In problem (P2), we calculated fuel cost from the
equations derived in Chapter 2. We will still use these
equations to correct for wind, but we must modify the distance
traveled in order to compute the fuel cost.

1. Obtain a feasible solution in the same way as before.
    This will give us an origin, destination, and
    refueling points in between.

2. Once we have these points, we will need to calculate
    the bearing from the origin to refueling point 1.

3. Obtain the aircraft speed. We can perform a
    regression on charts like those shown in figure 2-1
    in order to obtain a mathematical formula which gives
    us aircraft speed at the 99% max range given aircraft
    weight and altitude.

4. Obtain a vector of wind velocity, v, and wind
    direction, d, for all areas to be traveled through.
    It is preferable if these are uniformly located. The
    data that I used appear every 2.5 degrees north,
    south, east, and west for the entire world. This

basically made the earth a grid with a wind direction and velocity every 2.5 degrees.

5. Take the first segment traveled, from the origin to the first refueling point. Perform a mercator projection on these points. With a mercator projection, we can work in 2-dimensional space.

6. Find the equation of the line that runs through the origin and first refueling point. With this equation, we can find all points on the line from the origin to the first refueling point that touch our 'wind grid'.

7. Solve the equation for the line for all points that border on a grid line.

8. Convert all these points back to spherical coordinates.

9. Now, we still have our original line segment from the origin to the first refueling point, but it is broken into a bunch of smaller pieces according to the 'wind grid'. Take the first small segment from the origin. The end of this piece hits either a longitude or latitude multiple of 1.25n where n is odd (assuming the wind vectors are every 2.5 degrees). Calculate the distance of this small segment, t, and find the closest wind vector to it.

10. Take the bearing (Φ) from the origin to refueling point 1. This will not change while we are working on the smaller segments. Take the calculated aircraft speed, w, and get an aircraft velocity vector:

$$X = w \times \sin(\Phi)$$

$$Y = w \times \cos(\Phi)$$

11. Calculate a wind vector:

$$X' = v \times \sin(d)$$

$$Y' = v \times \cos(d)$$

11. Calculate a new direction vector:

$$X'' = X - X'$$

$$Y'' = Y - Y'$$

12. $w' = \sqrt{X''^2 + Y''^2}$

13. The new distance is:

$$d' = \frac{w}{w'} \times t$$

14. Calculate the fuel used based on d' and then calculate the 99% max aircraft speed.

15. Go to the next grid segment and keep repeating until you hit refueling point 1.

16. Sum all d' and the fuel used on each grid segment until refueling point 1. This is your fuel cost for the first segment.

17. Repeat this procedure for the next segment and keep
    repeating until at the destination. (i.e. refueling
    point 1 to refueling point 2) When you arrive at the
    destination, go back to the origin and repeat the
    process according to the algorithm in problem (P2).

Example Calculation

Origin = (W118, N36)

First Refueling Point = (W96, N49)
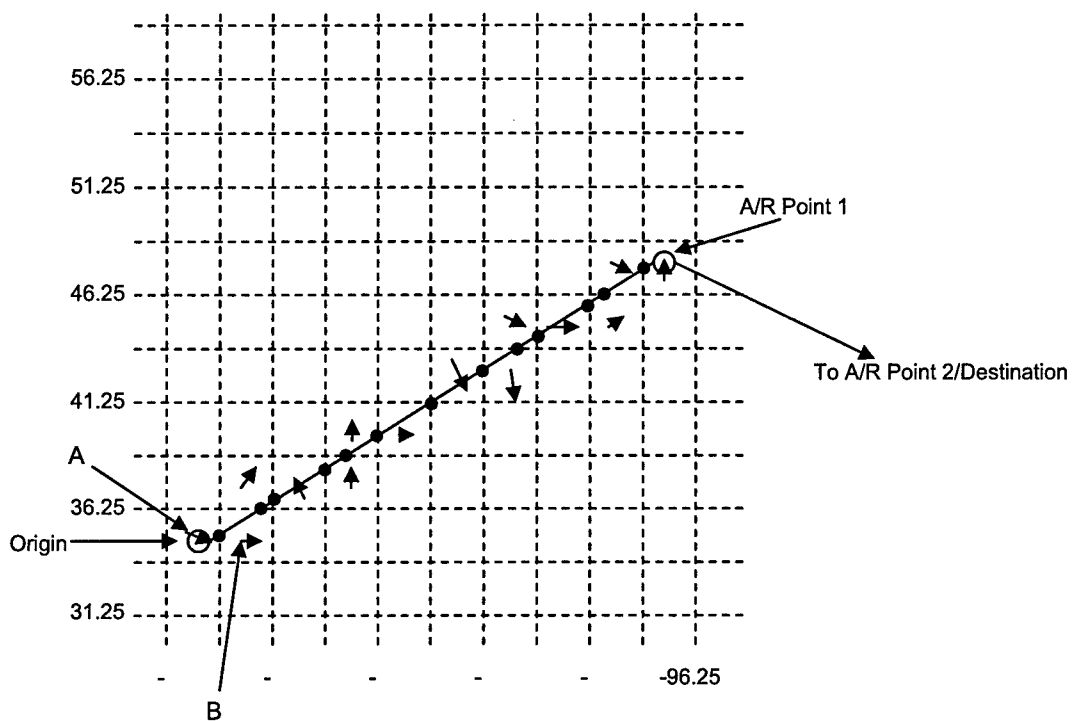
Figure 5-1. Problem P3 Illustration

In figure 5-1, the vector arrows within each grid box are the wind vectors associated that with particular portion of the grid. Note: the empty grid boxes also have associated wind vectors, but are not shown in Figure 5-1. The calculated bearing from the origin to aerial refueling point 1 is 44.196 degrees. Next, find the 99% max range speed the aircraft should travel at. For simplicity sake, we shall assume this to be 300 mi/hr for the first grid box we traverse, box A (although in the actual algorithm, one should compute the 99% max range speed based on aircraft weight and altitude). Next, convert the origin and aerial refueling point 1 into mercator points and we get:

Origin = [W118, N38.6331]

Point 1 = [W96, N56.368]

Next, use the equation of a line: y= mx+b. First, find m (the slope):

m = (y'-y)/(x'-x)=(-118+96)/(38.6331-56.3688061)=.8061

Now, plug in either the origin or point 1 to find b:

b = y-mx = 38.6331 - (.8061*-118) = 133.7567

Now, the equation of any point on the mercator map from the origin to point 1 is:

y = .8061x + 133.7567

We know we need to find the following points on the line:

x: [-116.25 -113.75 -111.25 -108.75 -106.25 -103.75

-101.25 -98.75 -96.25]

y: [36.25 38.75 41.25 43.75 46.25 48.75]

Plug the above points into the equation we found earlier and solve. Table 5-1 shows these points.

Table 5-1. Example Mercator Grid Lines

Longitude	Latitude
W118.0000	N38.6331 (origin)
W117.6161	N38.9426
W113.7064	N42.0944
W109.6572	N45.3585
W105.4500	N48.7501
W101.0632	N52.2865
W96.4715	N55.9879
W116.2500	N40.0439
W113.7500	N42.0592
W111.2500	N44.0745
W108.7500	N46.0899
W106.2500	N48.1052
W103.7500	N50.1205
W101.2500	N52.1358
W98.7500	N54.1512
W96.2500	N56.1665
W96.0000	N56.3680 (point 1)

Now, we must convert these points back to spherical coordinates, and sort them from the closest to farther points from the origin. These points are listed in table 5-2.

Table 5-2. Example Spherical Grid Lines

Longitude	Latitude
W118.00	N36.00 (origin)
W117.62	N36.25
W116.25	N37.13
W113.75	N38.72
113.71	N38.75
W111.25	N40.28
W109.66	N41.25
W108.75	N41.80
W106.25	N43.28
W105.45	N43.75
W103.75	N44.73
W101.25	N46.15
W101.06	N46.25
W98.75	N47.52
W96.47	N48.75
W96.25	N48.87
W96.00	N49.00 (point 1)

Now, assume the associated wind vector in box A is 35 mi/hr at
a direction of 120 degrees (0 degrees is due north). The
length of the segment in box A is simply the great circle
distance from the origin, (-118 36), to
(-117.62 36.25). This is 27.5 miles. The aircraft velocity
vector is:

    x = 300*sin(44.196) = 209.13

    y = 300*cos(44.196) = 215.09

The wind velocity vector is:

$x' = 35*\sin(120) = 30.31$

$y' = 35*\cos(120) = -17.50$

Now, we would like to keep our original velocity vector, so taking the wind vector into account, we get:

$x'' = 209.13 + 30.31 = 239.44$

$y'' = 215.09 - 17.50 = 197.59$

Next, compare the new magnitude, $\sqrt{x''^2 + y''^2} = 310.44$, to our original velocity magnitude, 300. Since 310.44>300, this route with wind present is going to take less effort than without the wind present. Thus, to account for the wind, we take the ratio of the original velocity to the wind factored velocity and apply this to the distance and get:

$(300 \div 310.44) \times 27.5 = 26.57$. Now, calculate the amount of fuel used for a distance of 26.57 miles. Subtract this amount from the original fuel allocation for the aircraft and then compute the new 99% max range speed for the next grid box, B. Repeat these steps to figure out the wind adjusted distance for grid box B and then repeat for each successive grid box until aerial refueling point 1 is reached. When aerial refueling point 1 is reached, we will have a fuel cost for leg 1, and then move onto the leg 2 and repeat all the aforementioned steps. Repeat this procedure for all legs of the routing for both tanker and deployment aircraft to get a total fuel cost.

With the wind adjusted total fuel cost, one then just uses the algorithm described in problem (P2) and keeps recomputing the wind adjusted total fuel cost as aerial refueling points are moved in conjunction with running that algorithm.

CHAPTER VI

ANALYSIS OF THE ALGORITHM

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1 Aircraft

N Aerial Refuelings

Non-Instantaneous Refueling

Comparison of Algorithm vs Current Planning Tools

Currently, the U.S. Air Force uses the Combined
Mating and Ranging Planning System (CMARPS) to plan aerial
refueling missions. For the most part, this tool directly
routes any deploying aircraft onto their great circle routes
(provided no restricted airspace is infringed upon during this
route), thus, minimizing the distance traveled by that
aircraft. It then places any needed aerial refuelings on this
great circle route according to refueler and abort base
availability. The algorithms derived in this paper use a
safety stock of fuel to deal with emergency situations rather
than specifically looking at distances to abort bases. Due to
this fact, we will also examine the effect on total fuel
consumption of using different amounts of safety stock at the

end of this chapter. The U.S. Air Force has provided 3 sample

missions they planned using CMARPS. The U.S. Air Force

mission details are in Tables 6-1, 6-2, 6-3, and 6-4. In all

scenarios, the deployment aircraft is a C-5 and the aerial

refueling aircraft are KC-10's.

Table 6-1: Base Information.

Mission	Origin	Destination	Aerial Refuelings	A/R Base 1	A/R Base 2
1	N39.133 W75.467	S31.933 E115.966	2	N38.266 W121.933	N13.583 E144.933
2	N38.266 W121.933	S31.933 E115.966	2	N38.266 W121.933	N13.583 E144.933
3	N33.88 W117.258	N9.516 E44.083	2	N47.966 W97.40	N63.983 W22.60

Table 6-2: CMARPS Data Results.

Mission	Offload #1 (lbs)	Offload #2 (lbs)	A/R Point 1 Beginning	A/R Point 2 Beginning	Total Fuel Consumption (lbs)
1	193,100	153,000	N50.233 W122.60	N21.50 E166.33	755,657
2	139,500	90,000	N32.66 W143.633	N1.067 E166.483	657,505
3	214,700	193,500	N45.033 W109.60	N66.366 W13.033	597,576

Table 6-3: CMARPS Fuel Consumption.

Mission	C-5 Fuel Consumption (lbs)	KC-10 #1 Fuel Consumption (lbs)	KC-10 #2 Fuel Consumption (lbs)
1	563,252	72,683	119,722
2	445,799	106,261	105,445
3	496,432	61,085	40,059

Table 6-4. Mission Parameters

Mission	C-5 Cargo Weight (lbs)	C-5 Initial Fuel (lbs)	KC-10 #1 Initial Fuel (lbs)	KC-10 #2 Initial Fuel (lbs)
1	100,000	267,150	310,000	330,000
2	100,000	267,150	260,000	260,000
3	190,000	114,150	330,000	280,000

Now, using the same input parameters given above, let us examine the solution for each mission using the algorithm derived in this paper. These results are shown in Tables 6-5 and 6-6.

Table 6-5: Algorithm Data Results.

Mission	Offload #1 (lbs)	Offload #2 (lbs)	A/R Point 1 Beginning	A/R Point 2 Beginning	Total Fuel Consumption (lbs)
1	188,010	135,129	N48.18 W120.88	N14.74 E147.19	656,971
2	92,089	127,057	N26.35 W156.08	N11.71 E146.04	618,679
3	232,597	186,851	N43.37 W110.68	N65.64 W21.35	595,236

Table 6-6: Algorithm Fuel Consumption.

Mission	C-5 Fuel Consumption (lbs)	KC-10 #1 Fuel Consumption (lbs)	KC-10 #2 Fuel Consumption (lbs)
1	563,797	61,829	31,345
2	456,933	133,432	28,313
3	506,019	62,958	26,259

As one would expect, the fuel savings using the algorithm described in this paper come from better placement of the refueling aircraft (the KC-10's). The C-5 burns more fuel using the described algorithm than CMARPS due to the fact that it is not possible for the C-5 to take a shorter route than the one prescribed by CMARPS.

Table 6-7 and Figure 6-1 show the affect of varying the reserve fuel requirement on the first mission. As one would expect, more fuel held in reserve drives the total fuel consumption higher as this forces the tanker aircraft to travel greater distances during the mission.

Table 6-7: Effect of Reserve Fuel on Fuel Consumption (Mission 1).

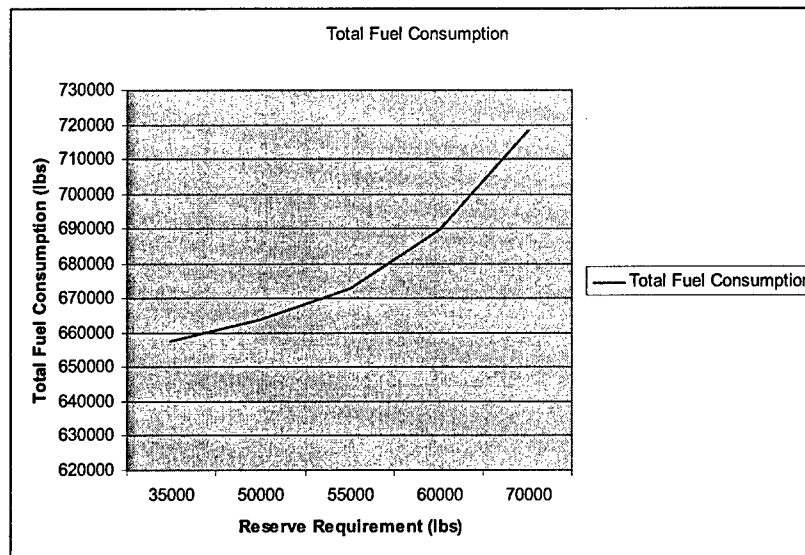| Reserve Fuel Requirement (lbs)	Total Fuel Consumption (lbs)
35,000	657,626
50,000	663,649
55,000	672,739
60,000	689,626
70,000	718,387

Figure 6-1: Reserve Fuel vs Safety Stock.

The overall performance of the algorithm holds up quite well in these examples. In every case, it outperformed the current method of mission planning. Table 6-8 shows the actual comparisons via mission. On average, the algorithm saved 6.96% in fuel consumption on a given mission.

Table 6-8: Comparison.

Mission	Algorithm Fuel Consumption (lbs)	CMARPS Fuel Consumption (lbs)	Fuel Savings (lbs)	Percentage of Savings
1	656,971	755,657	98,686	13.06%
2	618,679	657,505	38,826	5.91%
3	595,236	597,576	2,341	0.39%

Observation of an Additional Decision Variable

When running through the scenarios described above, it is apparent that the 2 decisions variables are:

1. What point to refuel at?

2. How much to offload at that point?

However, if we look at the fuel used and offloaded by each KC-10 above, we can come up with another decision variable that could possibly save fuel and money:

3. How much fuel to initially have in each KC-10?

Looking at our results, it is clear that in all cases, we can start with less fuel in the refueling aircraft. Let's assume that 30,000 pounds is the reserve requirement for the KC-10's. Therefore, the refueling aircraft must land with this amount of fuel. Table 6-9 shows the excess fuel we are carrying for each KC-10.

Table 6-9: KC-10 Fuel Utilization.

Mission	KC-10	Initial Fuel	Fuel Consumed	Fuel Offloaded	Fuel Left
1	1	310,000	61,829	188,010	60,161
	2	330,000	31,345	135,129	163,525
2	1	260,000	133,432	92,089	34,479
	2	260,000	28,313	127,057	104,630
3	1	330,000	62,958	232,597	34,445
	2	280,000	26,259	186,851	66,890

Table 6-10: New Initial KC-10 Fuel Allocations.

Mission	KC-10	Initial Fuel	Fuel Remaining	Excess Fuel	New Initial
1	1	310,000	60,161	30,161	277,050
	2	330,000	163,525	133,525	195,800
2	1	260,000	34,479	4,479	254,400
	2	260,000	104,630	74,630	185,000
3	1	330,000	34,445	4,445	325,500
	2	280,000	66,890	36,890	241,900

Table 6-11 shows the results for each mission using the new initial fuel for the KC-10's under the enhanced algorithm column. This enhancement slightly increases efficiency and boosts the fuel savings to 7.28% from the current system.

Table 6-11: Enhanced Algorithm Comparison.

Mission	Algorithm Fuel Consumption (lbs)	CMARPS Fuel Consumption (lbs)	Enhanced Algorithm Fuel Consumption (lbs)	Fuel Savings (lbs)	Percentage of Savings
1	656,971	755,657	653,451	102,206	13.52%
2	618,679	657,505	617,130	40,375	6.14%
3	595,236	597,576	593,850	3,726	0.62%

If we were to assume that a gallon of jet fuel costs the U.S. government $1.50 per gallon and one gallon is 6.8 pounds, the savings from using this algorithm on these three missions alone would be over $32,274. Assuming that these missions are a fraction of the daily workload for the Air Force, the potential cost savings over a year could be substantial.

CHAPTER VII

FUTURE RESEARCH

M Aircraft

N Aerial Refuelings

Non-Instantaneous Refueling

Future Research

Future research in this area should expand to include the option of more than one deployment aircraft per mission. This would allow fighter aircraft deployments to be optimized. Additionally, we could solve the problem of deploying two distinct types of aircraft, and thus two different fuel consumption functions, deploying from base A to base B with one or more aerial refuelings. Another possible problem to be solved would be the case where there are aircraft at base A, aircraft at base B, and they are both deploying to base C. En route to base C, these aircraft will require aerial refueling from predetermined refueling bases. This problem entails finding a procedure that finds the near-optimal refueling points with more than one source. Ultimately, one could solve the mixture of the two aforementioned problems: finding

optimal refueling points for different types of aircraft at
different bases deploying to the same base with predetermined
refueling bases to be utilized.

Finally, in order to add even more reality to the model,
I think it may be wise to incorporate flight restrictions into
the model. For example, the U.S. Air Force can not currently
plan flight routes through certain countries due to political
considerations (i.e. North Korea, Iran, etc.). However, a
minimal cost path may be found through these countries since
the aforementioned algorithms do not recognize political
boundaries. Therefore, to make the algorithm more useful, it
would make sense to be able to add flight restrictions.

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